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TEMPORAL-SPATIAL COMPRESSION ANTENNAS

Paul VanEtten

Rome Air Development Center  
Griffiss Air Force Base, New York

June 1973

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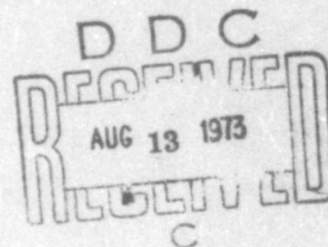
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## FOREWORD

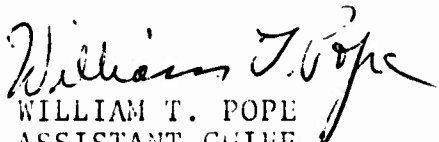
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This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Office (NTIS).

This technical report has been reviewed and is approved.

  
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## ABSTRACT

The idea of using an antenna as a pulse compression network is presented employing the natural dispersion inherent in some broadband antennas. It is shown that the power densities resulting from such antennas are increased by the time-bandwidth product of the antenna over its "CW" power density.

Experimental measurements have been performed on two (2) antennas to determine their transfer functions. Both the circular-polarized cavity-backed spiral and the linearly-polarized pyramid log periodic antennas have excellent chirp transfer functions and the results are given.

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## SECTION I

### INTRODUCTION

The Signal Radiation and Processing Section of the Techniques Branch at Rome Air Development Center (RADC) has been involved in an effort called "Impulse Technology." In this effort, very short pulses in the picosecond range, and extremely large bandwidths approaching 8 GHz, are being utilized.

The most difficult problem encountered thus far is due to the antenna. In the past, efforts were directed to the problem of transmitting and receiving a very fast rise time short pulse (approaching an impulse) without introducing dispersion into the system. This dispersion, or antenna ringing, is produced by the antenna when hit with an impulse type waveform. The three (3) major design parameters of impulse antennas are directivity, signal fidelity, and efficiency. In general, antennas with good directivities and efficiencies have dispersion leading to severe signal distortion.<sup>1</sup>

It appears that a natural solution to this problem is to employ the dispersion in an antenna system thus maximizing the desired parameters. It is the object of this report to consider the antenna system as a pulse compression network which is being fed by a chirp waveform transmitter. This will result in an impulsive waveform which is formed spatially.

## SECTION II

### BASIC THEORY

To better understand temporal-spatial compression a review of simple antenna theory is appropriate. Refer to Figure 1. Here there are four (4) configurations, a through d, of transmitter(s) and radiating element(s) with their respective field intensities and power densities. Figure 1-A shows a one watt transmitter feeding a one ohm transmission line producing one ampere of current flowing on a radiating element. At a given fixed range this will result in a field intensity of  $e_1$  and a power density of  $P_1$ .

Now consider arraying two elements in the conventional manner, as seen in Figure 1-B. Here the one watt transmitter has its power divided equally between the two radiators, producing a current flowing on each element of  $\frac{1}{\sqrt{2}}$  amperes. The field intensity at the same fixed range as above will be the vector sum of the emission from both radiating elements. Notice that the field intensity is the  $\sqrt{2}$   $e_1$  and the power density is  $2P_1$ . It is assumed here that no mutual coupling exists between the two radiating elements. In general, with an array of N elements the power density is simply N times the power density of each element.

In Figure 1-C, each element is fed with a one watt transmitter. At the same fixed range, the field intensity is again the vector sum of the two radiators, equal to  $2e_1$ . However, the corresponding power density  $P_d$  equals  $4P_1$ . For N radiators each being fed with a one watt transmitter, the power density is thus equal to  $N^2 P_1$ . Notice the power density increase over the conventional array of Figure 1-B. Here gain boosting may be implied,

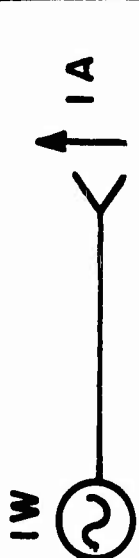
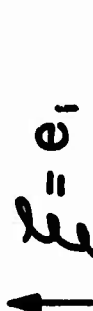
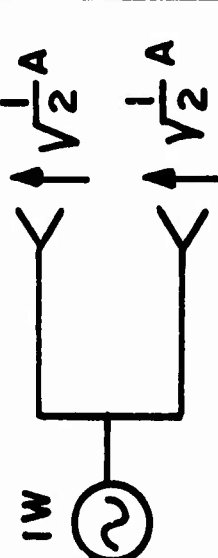

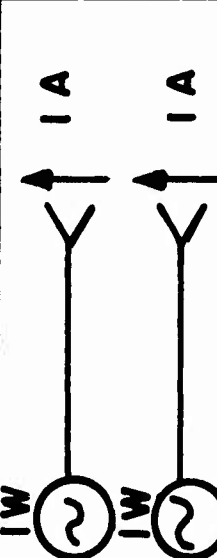

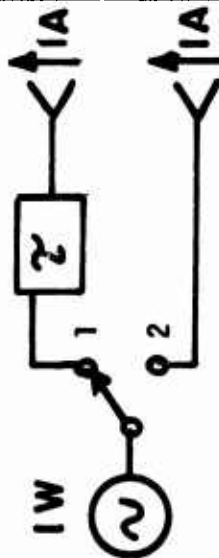
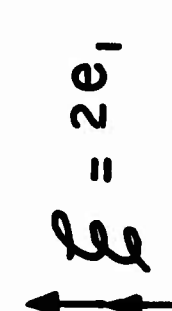
Array	Field Intensity	Power Density
		$P_d = \frac{e_1^2}{120\pi} = P_1$
		$P_d = \frac{2 e_1^2}{120\pi} = 2P_1$
		$P_d = \frac{4 e_1^2}{120\pi} = 4P_1$
		$P_d = \frac{4 e_1^2}{120\pi} = 4P_1$

FIGURE 1: Various Combinations of Transmitters and Radiators with Their Respective Field Intensities and Densities

Figure 1a

1b

1c

1d

however the total transmitter power was increased by a factor of  $N$ , only.

The idea of time sorting will now be introduced. What is required with time sorting is maintenance of the power density of Figure 1-C, but employing only one transmitter instead of  $N$  transmitters. A one watt transmitter may be time switched between two elements, one with a delay medium giving  $T$  seconds delay, as seen in Figure 1-D. In this illustrative example the time sorting switch operates every  $\tau$  seconds. If the one watt transmitter provides a CW waveform then the waveform feeding each radiating element is as shown in Figure 2. Here, for one half of the time, full power appears at both radiating elements giving a field intensity of  $2e_1$  at the same range mentioned previously, and the power density is thus  $4P_1$ . In this simple example the power density is twice that of a conventional array (Figure 1-B), but the gain boosting principle (i.e., the increased power density) applies for only one half of the time. This appears reasonable from the conservation of energy principle.

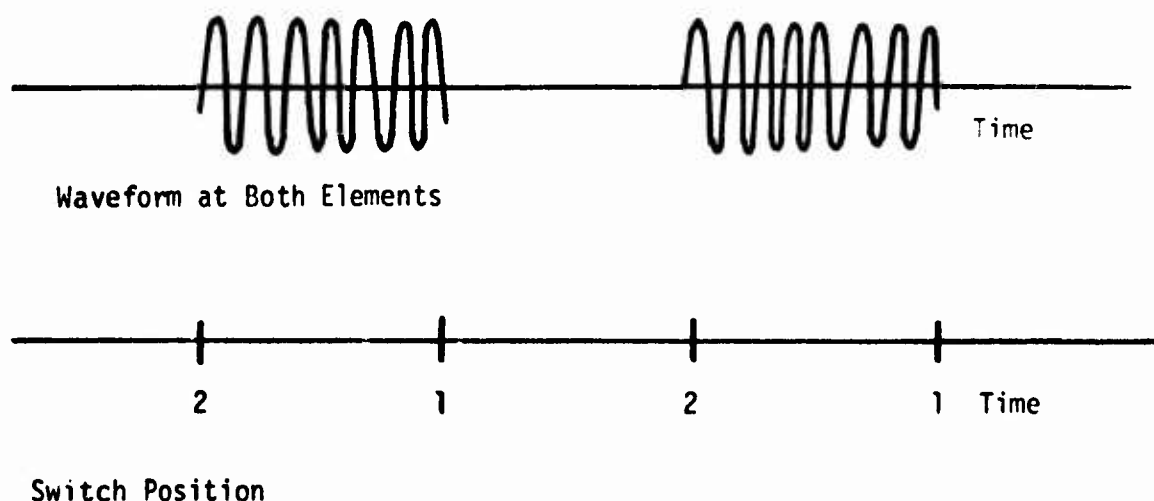


FIGURE 2: Waveform at Both Elements and Switch Position of Antenna System Shown in Figure 1

### GAIN BOOSTING EMPLOYING TIME SORTING

Gain boosting employing time sorting for an N element array is described with the aid of Figure 3.

Here for convenience of illustration, a CW transmitter is being time sorted among the N array elements. The antenna element No. 1 contains the maximum delay of  $N\tau$  and the delay decreases successively by  $\tau$  until the last array element has no relative delay. Notice here that use of time sorting and the delay mediums allows the transmitter power to "load up" full power across the entire array. If the time sorting switch is allowed to continue to switch at a rate of  $\tau$  per position, the waveform appearing instantaneously on all elements is seen in Figure 3-B. The power density of the time sorting array is equal to  $N^2$  times the power density of a single radiating element as given by the equation:

$$P_d = \frac{P_T N^2 G (\text{element})}{4\pi r^2} \quad (1)$$

It is seen by inspection of Figure 3-B that the high field densities, with gain boosting, occurs only for  $1/N$  part of the time. It may be said, therefore, that this time sorting principle is achieving gain boosting over a time period which is less than the time waveform emitted from the conventional type of antenna.

### TIME-FREQUENCY DECODER MATRIX

In the above time-sorting operation a switch was employed at the output of the transmitter. This may be undesirable for some applications

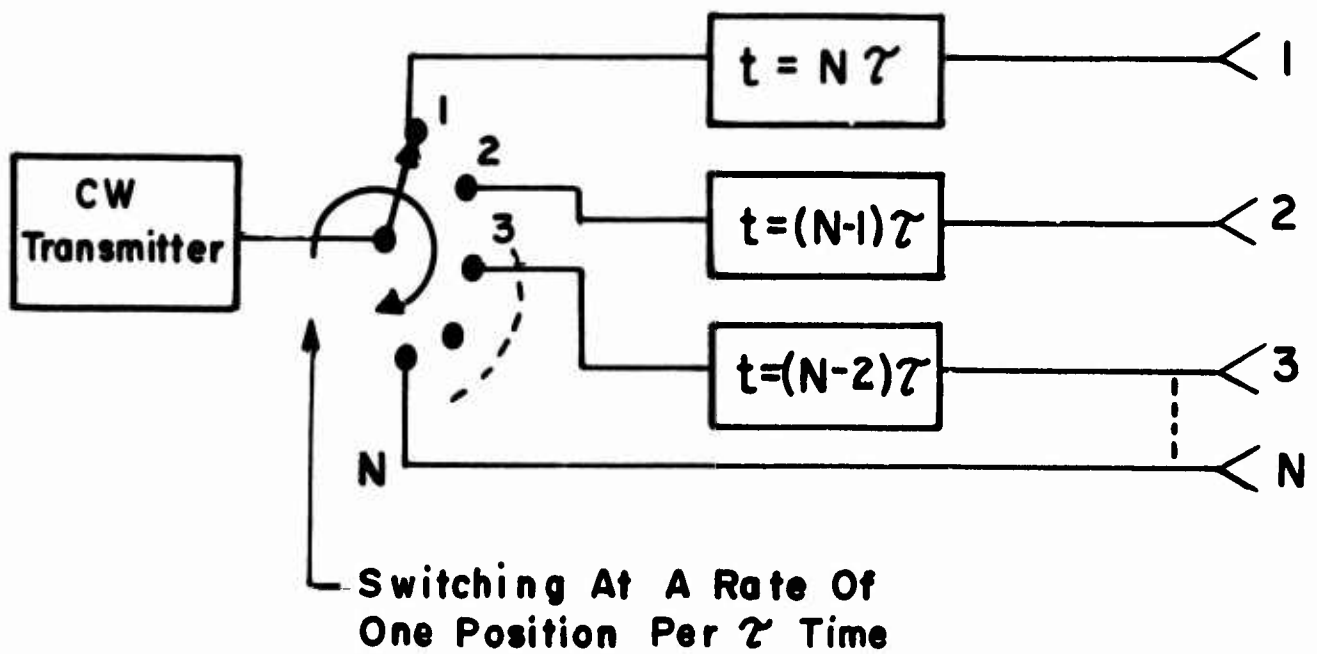


FIGURE 3-A: Block Diagram of an "N" Element Time Sorting Array

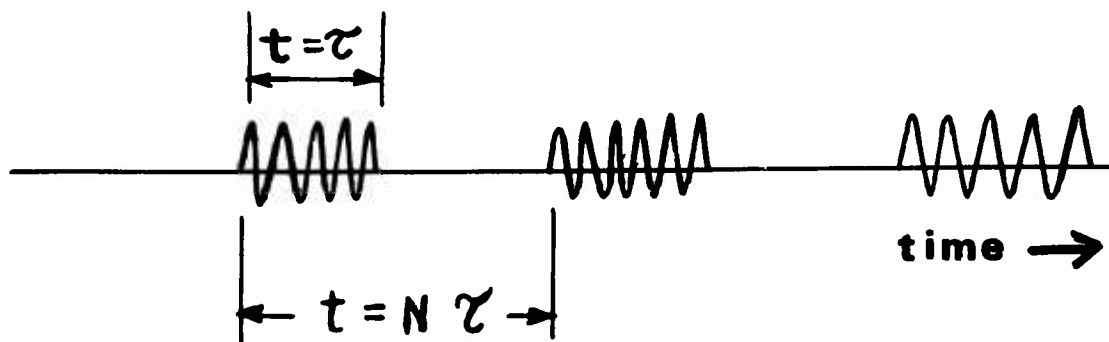


FIGURE 3-B: Time Waveform at Each Antenna Element

(e.g., when high transmitter power is employed.) To circumvent this problem a second principle will be explained. This principle employs a time-frequency decoder matrix, as shown in Figure 4. For ease of explanation a frequency coded pulse with a waveform as shown in Figure 5 will be assumed. The long burst contains N sub-bursts of frequency coded waveforms of frequency  $F_1, F_2, \dots, F_N$ . A restriction on the waveform is that the spectrum of each sub-burst shall not overlap the spectrum of any other sub-burst.

The coded waveform of Figure 5 feeds the decoder matrix, as shown in Figure 4. The bandpass filters and delay networks are so adjusted that the sub-bursts will arrive at each of the N elements at the same time. Each sub-burst must be properly phased, such that coherent vector addition will occur in far-field, thereby obtaining gain boosting in the same manner as Figure 1-D and Figure 3. The waveforms occurring simultaneously at the antenna elements are shown in Figure 6. It is a requirement that the time-bandwidth product of the coded waveform from the transmitter must be equal to or greater than the gain of the array without gain boosting. Here again, as in the system shown in Figure 3, gain boosting is achieved, with higher peak power densities obtained at the expense of radiating a waveform of shorter time duration.

The analytic expression for the set of N waveform bursts in which all amplitudes are assumed to be equal to  $1/N$  and all phase angles equal to zero is:

$$f(t) = \frac{1}{N} [\cos(w_1 t) + \cos[w_1 + \Delta w] t + \cos[w_1 + 2 \Delta w] t \dots + \cos[w_1 + (N - 1) \Delta w] t] \quad (2)$$

where  $w_1 = 2\pi f_1$



# Gain Boosting Employing Temporal- Spatial Integration

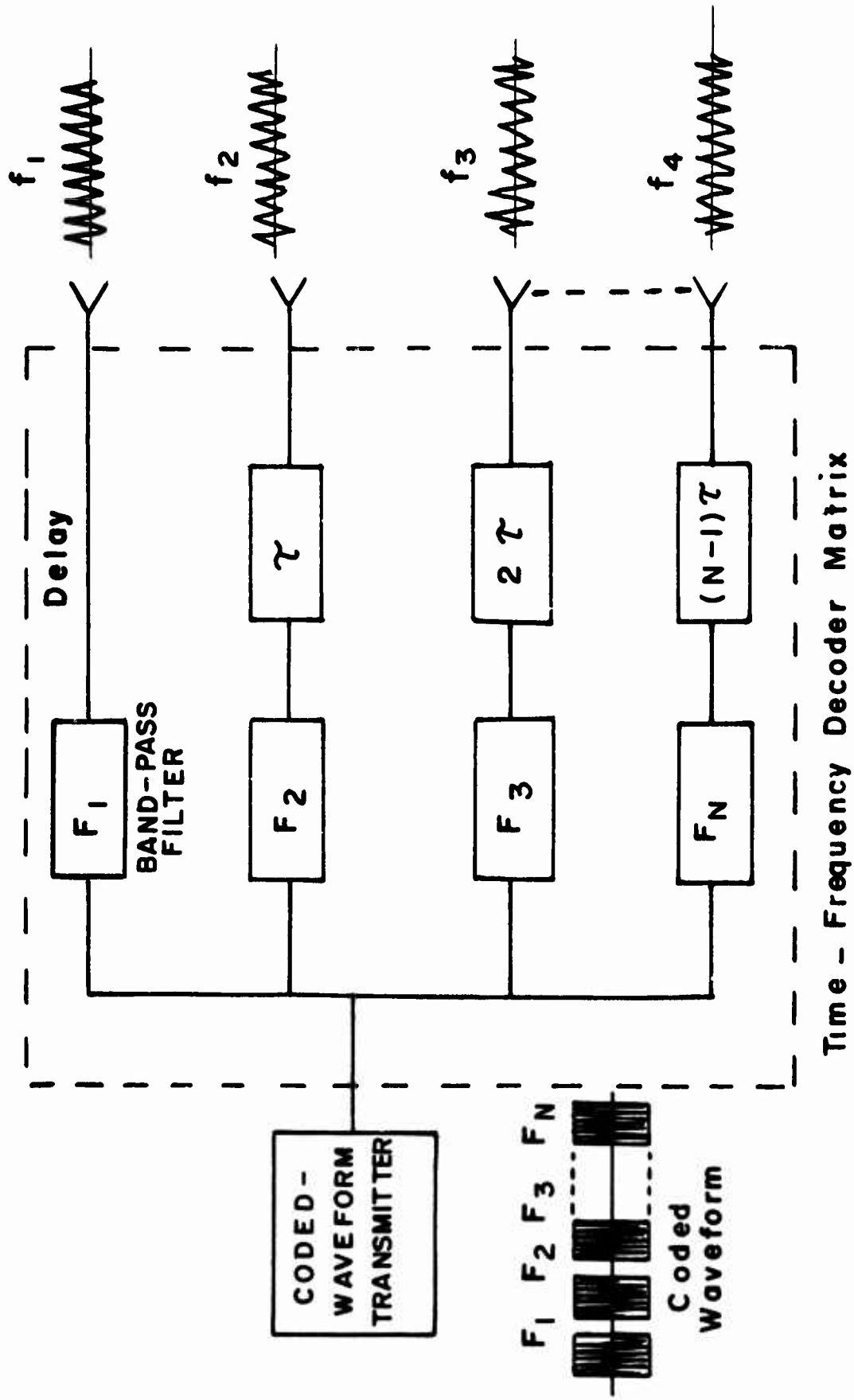


FIGURE 4: Block Diagram of an "N" Element Antenna

Amplitude

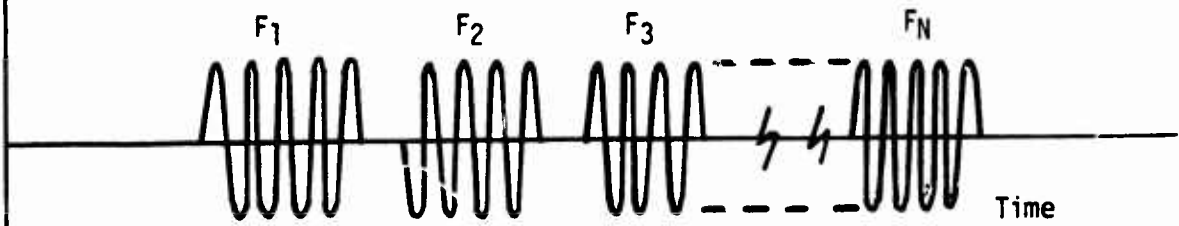


FIGURE 5: Coded Waveform from Transmitter Feeding the Time-Frequency Decoder

Waveforms to  
Antenna Elements #

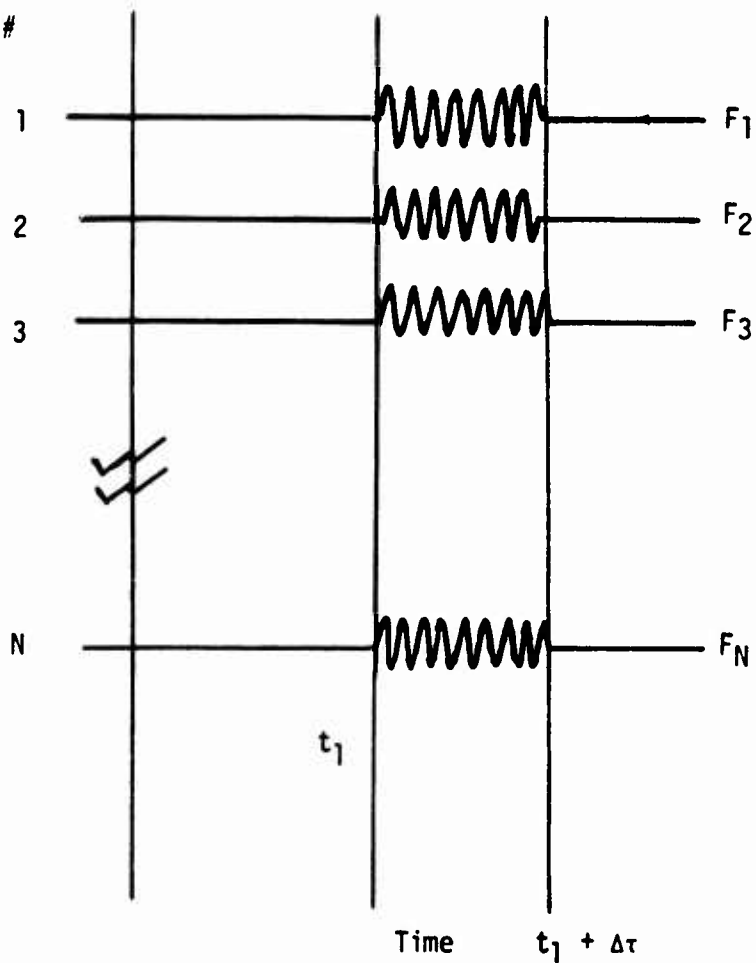


FIGURE 6: Waveforms Arriving at Antenna Elements

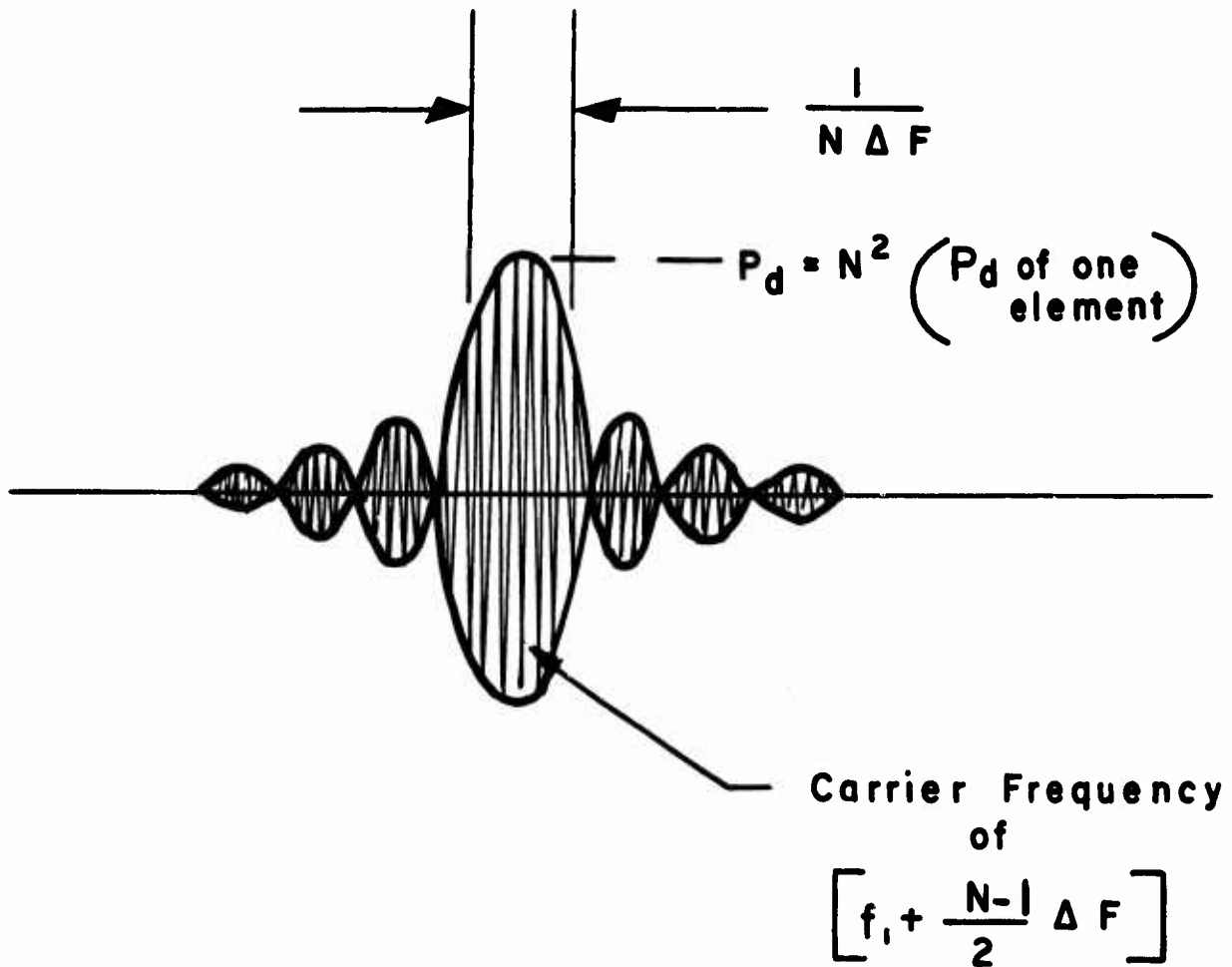


FIGURE 7: Time Waveform of the Spatial Integrated Field Intensity of N Radiating Elements

This can be expressed in closed form:

$$f(t) = \frac{\sin \left[ \frac{N \Delta \omega t}{2} \right]}{N \sin \left[ \frac{\Delta \omega}{2} t \right]} \cos \left[ \omega_1 + \frac{(N-1) \Delta \omega}{2} t \right] \quad (3)$$

This waveform is shown in Figure 7. Other types of coded waveforms may be employed, but the one above was used for illustrating the time-frequency decoder matrix. Notice that the spatial waveform has been compressed compared to the transmitter's waveform, by an amount equal to the transmitter's time-bandwidth product.

### SECTION III

#### SPATIAL FIELD COMPRESSION

In Section II a physical matrix was employed to obtain a large time-bandwidth product. Although the principle is sound, hardware (i.e., delay medias and bandpass filters) is required which may become bulky and costly. The idea of using an antenna with natural built-in dispersion appears to be a logical solution for obtaining enhanced electromagnetic fields.

To understand how an antenna may have a natural dispersion or large time-bandwidth product, a coplanar log-periodic antenna is used as an example (see Figure 8). Log-periodic antennas are always fed such that the highest frequency element is excited first, followed by the next lower frequency element and so on. If the radiating elements are spaced a distance of  $\ell$  apart, the delay of the driving signal between elements is

$$\tau_1 = \frac{\ell}{V} \quad (4)$$

where  $V$  is the propagation velocity on the feed. The total delay imposed upon the driving signal by the feed between element  $f_1$  and the last element  $f_N$  is:

$$\tau_d = (N - 1) \frac{\ell}{V} \quad (5)$$

In addition to the feed delay which is dependent on the velocity of propagation of the feed structure, additional time delay is encountered upon radiation. This is obvious when one considers that upon radiation from the last element,  $f_N$ , the radiated waveform travels an additional distance of

$(N - 1) \ell$  relative to the radiated signal from  $f_1$  for forward radiation.

The additional time delay is

$$T_S = (N - 1) \frac{1}{C} \quad (6)$$

where  $C$  is the velocity of propagation in free space. On boresight, i.e., forward radiation, the total time delay between element  $f_1$  and  $f_N$  in the far field is:

$$T_T = (N - 1) \left( \frac{\ell}{C} + \frac{\ell}{V} \right) \quad (7)$$

If the velocity,  $V$ , of the feed transmission line is  $C$ , the total delay is:

$$T_T = (N - 1) \frac{2\ell}{C} \quad (8)$$

One can now consider feeding the antenna with a chirp pulse which its instantaneous frequency is increasing from  $f_N$  to  $f_1$ . The time duration of the frequency modulated pulse is  $\tau$ . The manner in which the instantaneous frequency is swept from  $f_N$  to  $f_1$  may be linear, logarithmic or any other function so long as it is the conjugate of the time delay/frequency relationship of each element discussed in the previous paragraphs. That is to say, that in the far field; the peak of a cycle of RF of  $f_1, f_2, \dots, f_N$  will all arrive at the same time and be in phase. This will result in pulse compression performed spatially similar to a pulse compression network. The total electromagnetic field will be the sum of each element:

$$\xi_T = \sum_1^N \xi(\text{element}) \quad (9)$$

and for equal gain elements (e.g., one-half wavelength dipoles). The electromagnetic field is:

$$\xi_T = N \xi_{(\text{element})} \quad (10)$$

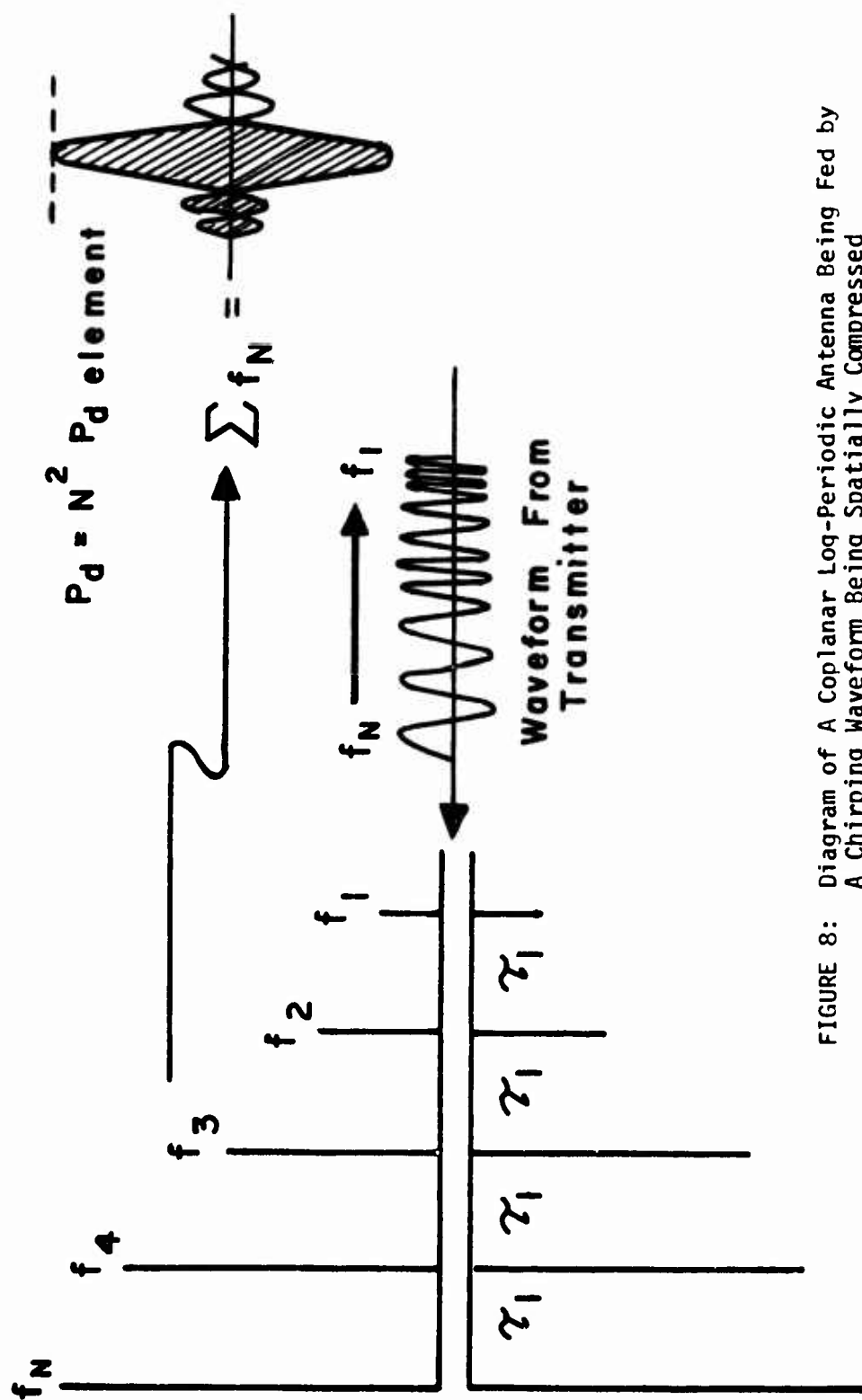


FIGURE 8: Diagram of A Coplanar Log-Periodic Antenna Being Fed by A Chirping Waveform Being Spatially Compressed

It follows then, that the total power density is:

$$P_d = N^2 P_d (\text{element}) \quad (11)$$

It should be noted that full transmitter power is fed to each element and it is for this reason that the apparent antenna gain is greater than the gain of a conventional antenna of the same size.

Because of spatial compression the power density of the compressed electromagnetic wave ( $P'_d$ ) is:

$$P'_d = BT P_d \quad (12)$$

where B is the bandwidth of the chirp waveform, T is the time duration of the F.M. pulse and  $P_d$  is the power density of the antenna when radiating CW. The restriction

$$BT \geq G_{CW} \quad (13)$$

applies at all times.

The above discussion provides a physical interpretation of the idea of spatial field compression. The following section will present a rigid derivation of the performance of spatial field compression from the point of view of antenna transfer functions.

## SECTION IV

### DERIVATION OF PERFORMANCE EQUATIONS

In this section performance equations will be derived for two different cases. The first case is where only the transmitting antenna is optimized to the transmitter to obtain an enhanced field. The second case employs both a transmitting and receiving antenna such that the waveform at the receiving antenna terminals is optimized.

It is appropriate to define the antenna's transfer functions and notations. Defining the transmitting transfer function,  $H_T (w)$ , of an antenna as

$$H_T (w) = \frac{\Xi (w)}{E_T (w)} \quad (14)$$

where  $\Xi (w)$  is the electromagnetic field intensity at a range  $R$  and  $e(t)$  is the exciting voltage to the antenna. It is shown in Appendix "A" that:

$$H_T (w) = \frac{\sqrt{30 G (w)}}{R \sqrt{Z_a (w)}} \quad (15)$$

where  $G$  = Antenna Gain

$R$  = Range

$Z_a$  = Antenna Input Impedance.

The definition of the receiving transfer function of an antenna,  $H_R (w)$ , is:

$$H_R (w) = \frac{E_R (w)}{\Xi (w)} \quad (16)$$

where  $E_R (t)$  is the voltage received across the antenna terminals and  $\Xi (w)$  is the field intensity at the receiving aperture.



It is shown in Appendix "B" that:

$$H_R (w) = \sqrt{\frac{A (w) Z_a (w)}{120\pi}} \quad (17)$$

where  $A (w)$  is the antenna's aperture.

The relationship<sup>2,3</sup> between the transmitting and receiving transfer functions is (derived in Appendix "C"):

$$h_T (t) = \frac{d [h_R (t)]}{dt} \equiv h_R' (t) \quad \text{or} \quad (18)$$

$$h_R = \int_{-\infty}^{+\infty} h_T (t) dt. \quad (19)$$

That is to say the transmitting transfer function is the derivative of the receiving transfer function.

Also the notations  $h (t) \Rightarrow H (w)$

$\xi (t) \Rightarrow \Xi (w)$

$e (t) \Rightarrow E (w)$

are used where  $\Rightarrow$  denotes the Fourier transform.

#### CASE I OPTIMIZING THE RADIATED ELECTROMAGNETIC FIELD:

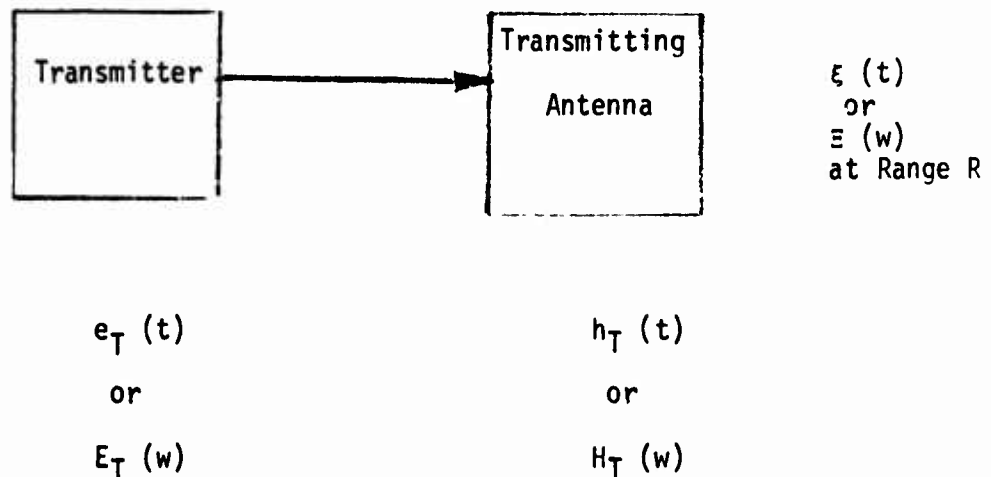


FIGURE 9: Block Diagram of Transmitter and Transmitting Antenna Showing Waveform Notations.

The condition here is to optimize the electromagnetic field,  $\xi(t,R)$  as seen in Figure 9. If the transmitter has a waveform  $e_T(t)$  and the antenna has a transfer function  $h(t)$  then the relationship of the two are adjusted such that antenna's transfer function is the complex conjugate of the transmitter's waveform.

$$\text{That is:} \quad h_T(t) = K e_T^*(t) \quad (20)$$

where  $K$  is a normalizing factor.

The equation for the field intensity is

$$\Xi(w) = E_T(w) H_T(w) \quad (21)$$

Considering the antenna to be a matched filter and normalizing maximum filter gain to unity<sup>4</sup>:

$$\frac{1}{\sqrt{P_T Z_a}} E_T^*(w) = \frac{R \sqrt{Z_a(w)}}{\sqrt{30 G(w)}} H_T(w) \quad (22)$$

The field intensity,  $\Xi(w)$ , can now be expressed:

$$\Xi(w) = \sqrt{P_T Z_a} E_T^*(w) \frac{\sqrt{30 G(w)}}{R \sqrt{Z_a(w)}} E_T(w) \quad (23)$$

$$\Xi(w) = \frac{\sqrt{30 P_T G(w)}}{R} E_T^*(w) E_T(w) \quad (24)$$

The  $E_T^*(w) E_T(w)$  product may be considered as  $E_T(w)$  passing through its matched filter. It is well known<sup>4, 5</sup>, for the case of linear frequency modulated pulses:

$$\text{Compression Ratio} \equiv C \cong BT \quad (25)$$

where  $T$  is the time duration of  $e_T(t)$  and  $B$  is the bandwidth of the

transmitter's waveform. Also

$$C \approx \frac{T}{\tau} \quad (26)$$

where  $\tau$  is the radiated waveform's pulsewidth, and the peak value of the waveform is increased by  $\sqrt{BT}$ .

For linear frequency modulation coding the term  $E_T^*(w) E_T(w)$  is constant with frequency and also normalized to one, therefore:

$$\Xi(w) = \frac{\sqrt{30 P_T G(w) BT}}{R} \quad (27)$$

Since:

$$P_d = \frac{\Xi^2(w)}{120\pi} \quad (28)$$

Then:

$$P_d = \frac{P_T G B T}{4\pi R^2} = \frac{P_T G C}{4\pi R^2} \quad (29)$$

Notice that the peak power density has increased by  $C$ , the compression ratio. It can be said therefore that the peak effective radiated power density has been increased by a factor equal to the time-bandwidth product of the transmitter's waveform.

The above discussion considered obtaining an increased electromagnetic field with time-spatial compression, i.e., transmit only case. Next, consideration will be given to the transmit-receive antenna system.

CASE II OPTIMIZING THE RECEIVED WAVEFORM EMPLOYING BOTH A TRANSMITTING  
AND RECEIVING ANTENNA:

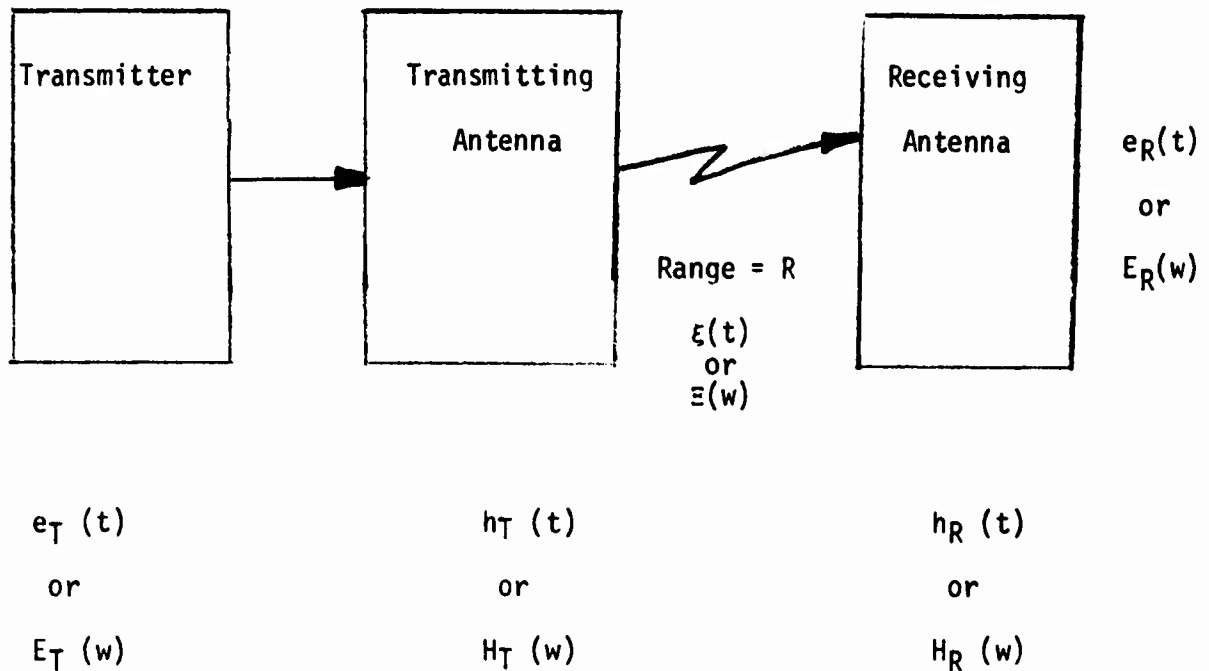


FIGURE 10: Block Diagram of Transmitter and Antennas Showing Time Waveform Notations.

In this case the received waveform,  $e_R(t)$ , as seen in Figure 10 is to be optimized.

The expression for the received waveform is:

$$E_R(w) = E_T(w) H_T(w) H_R(w) \quad (30)$$

In this case using two antennas compression will occur on both radiation and reception of the transmitted signal. To maintain perfect compression

the following condition must be maintained:

$$E_T^*(w) = H_T(w) H_R(w) \quad \text{or} \quad (31)$$

$$E_T(w) = H_T^*(w) H_R^*(w) \quad (32)$$

This is analogous to compressing a chirp pulse in two cascaded filters, the first filter partially collapsing the chirp waveform and the second filter completing the compression. Considering the antenna as a matched filter and normalizing maximum filter gain to unity (maintaining gains or attenuation constants):

$$\frac{1}{\sqrt{P_T Z_a}} E_T^*(w) = \frac{R \sqrt{Z_a}}{\sqrt{30G}} H_T(w) \sqrt{\frac{120\pi}{A Z_a}} H_R(w) \quad (33)$$

where  $G$  is the transmitting power gain,  $A$  is the receiving aperture, and  $Z_a$  is the antenna input impedance. The received waveform can now be expressed:

$$E_R(w) = \sqrt{P_T Z_a} E_T^*(w) \frac{\sqrt{30G}}{R \sqrt{Z_a}} H_T(w) \sqrt{\frac{A Z_b}{120\pi}} H_R(w) \quad (34)$$

$$E_R(w) = \frac{\sqrt{P_T G A Z_b}}{R \sqrt{4\pi}} E^*(w) H_T(w) H_R(w) \quad (35)$$

For linear frequency modulation coding the terms  $E^*(w) H_T(w) H_R(w)$  are constant with frequency and also normalized to one, therefore:

$$E_R(w) = \frac{\sqrt{P_T G A Z_b B T}}{R \sqrt{4\pi}} \quad (36)$$

The corresponding received peak power is:

$$P_R (w) = \frac{E_R^2 (w)}{Z_b} = \frac{P_T GA BT}{4\pi R^2} \quad (37)$$

Consider Case II employing identical antennas for transmitting and receiving using the relationship:

$$\frac{d [h_R (t)]}{dt} = h_T (t) \quad \text{or} \quad (38)$$

$$j\omega H_R (w) = H_T (w) \quad (39)$$

Recalling the requirement:

$$E_T^* (w) = H_R (w) H_T (w) \quad (40)$$

must be maintained for pulse compression.

From the above two equations it is seen that

$$E_T^* (w) = j\omega H_R^2 (w) \quad (41)$$

or

$$E_T^* (w) = \frac{1}{j\omega} H_T^2 (w) \quad (42)$$

Therefore, the transmitter waveform can be expressed as a function of either the transmitting or receiving transfer function.

## SECTION V

### EXPERIMENTAL MEASUREMENTS

Presented in this section is experimental data taken on the transient response of two different antennas. Both a linearly-polarized pyramidal log periodic antenna and a circular polarized cavity backed spiral antenna were measured for their dispersion properties.

In measuring the transient response of antennas it is useful to employ an antenna system having both the capability of radiating and receiving a time limited waveform, ideally an impulse, without distortion. A pair of properly designed monopoles (see Figure 11) provide this capability. The transmitting monopole is simply a coaxial cable (type RG 333/U) with the center conductor protruding approximately five (5) feet. Upon being fed with a 100 picosecond pulse, radiation occurs at the protrusion center and at the tip end of the center conductor. Since the monopole center conductor is five (5) feet long, the radiation from the tip will occur more than five nanoseconds later and can be time gated out. The receiving monopole is similar with the exception that the center conductor protrudes only one inch. In this experiment the impulse generator was IKOR's Model R100 delivering a 1000 volt, 100 picosecond pulse. For receiving, a Hewlett-Packard sampling scope Model 140A/1421A/1411A with the 1430A sampling head is connected directly to the receiving monopole with the appropriate attenuators. The received impulsive waveform is seen in Figure 12. The second pulse is the ground bounce which is also positive since the monopoles are vertically polarized. Although an impulsive signal can be radiated and received with little or no distortion employing the monopole antennas, the system is inefficient since the antennas exhibit a gain-aperture product of 46 dB below a square meter.



FIGURE 11: Photograph of the One Inch and Five Foot Monopoles - Mounted on Wooden Stands

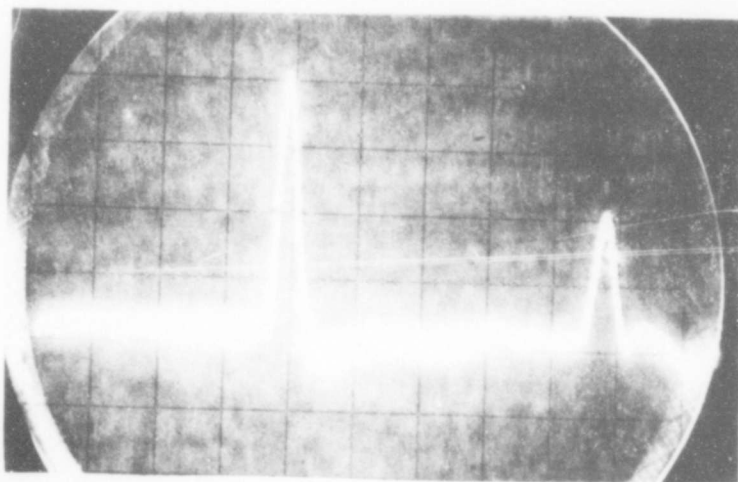


FIGURE 12: Response of Monopoles. Second Pulse is Ground Bounce. Sweep Speed is One Nanosecond Per Division



#### A. Log Periodic Measurements:

A pyramidal log periodic antenna (see Figure 13) was impulsed to obtain its transfer function. The test antenna is AEL Model APN-502A with published claims to operate over a frequency range of 0.2 GHz to 3 GHz with a VSWR of 2 and a gain of 11 dB. A 100 picosecond impulse is fed to the log periodic antenna while the receiving antenna, a one inch monopole, is connected to the H.P. sampling scope and is positioned twenty feet away from the test antenna. Over the frequency ranges of 3 GHz and less the one inch monopole can be considered a perfect receiving probe. For this reason the received waveform is considered as the transmitting transfer function if the antenna under test is fed with an impulse.

The response to the 100 picosecond impulse is seen in Figure 14. As characterized by dispersive antennas, the antenna under test radiates the highest "instantaneous frequency" first followed by succeeding lower instantaneous frequencies. The waveform of Figure 13 contains distortion at about 4 nanoseconds after the leading edge of the waveform because of ground reflections. Near the end of the sweep severe distortion can be observed which is due to reflection from nearby objects. Because of these reflections the response is only accurate for radiation from approximately the first half of the antenna.

A plot of "instantaneous frequency" vs time is shown in Figure 15. It is noted that the chirp characteristic is not linear. This is reasonable because of the logarithmic spacing of the elements and Figure 15 appears as a logarithmic function. Over the frequency range from 0.5 GHz to 5 GHz the dispersion time is approximately 6 nanoseconds. This results in a time-bandwidth product of 27.

Because of the reflections the response of the antenna down to 200 MHz with its corresponding dispersion could not be determined accurately. The total time-bandwidth product however would be greater. Since the CW gain of the antenna is 11 dB and the BT product is greater than 14 dB, the effective gain is greater than 25 dB.

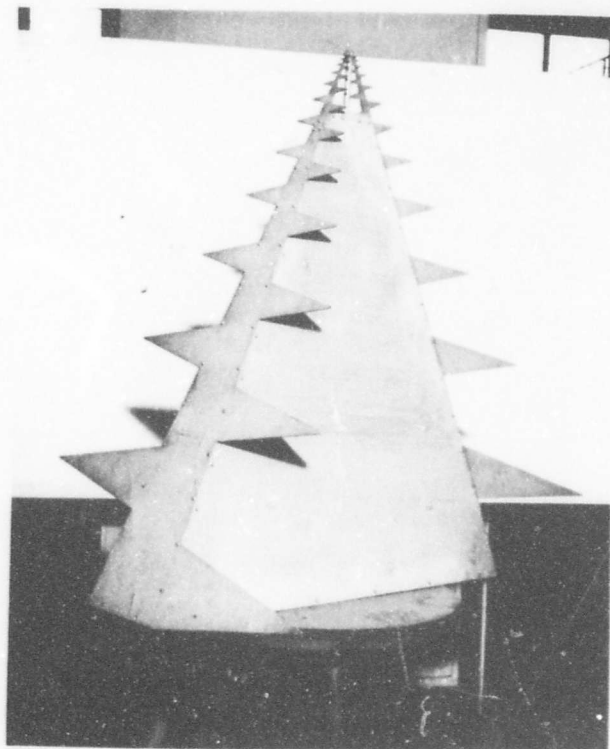


FIGURE 13: Photograph of Pyramidal Log Periodic Antenna AEL Model #APN-502A  
(Height of Antenna is 42 Inches)

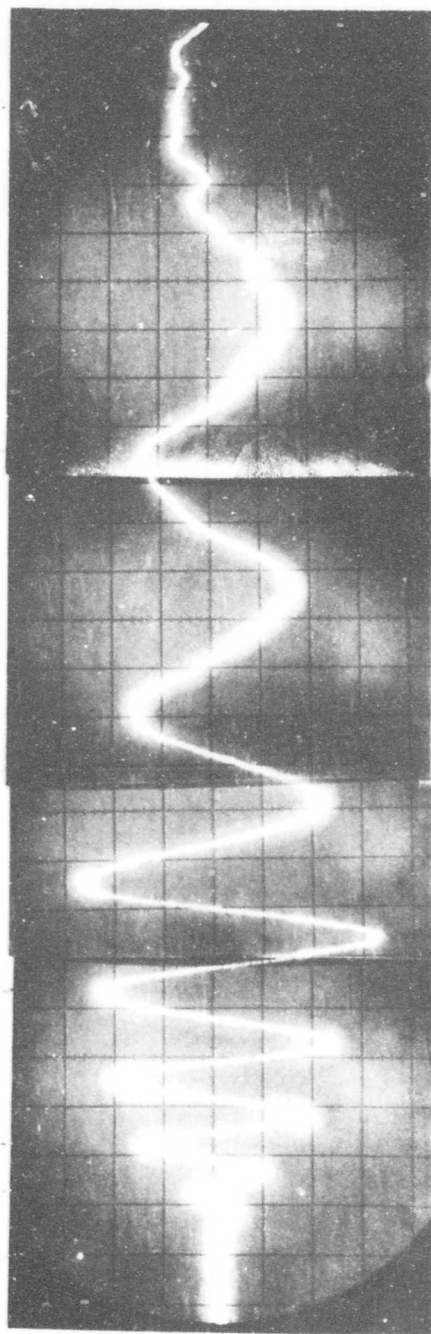


FIGURE 14: Transmitting Transfer Function Using a 100 psec Pulse For Excitation of Log-Periodic Antenna, Model APN-502A.

SWEEP RATE: 400 picoseconds per division

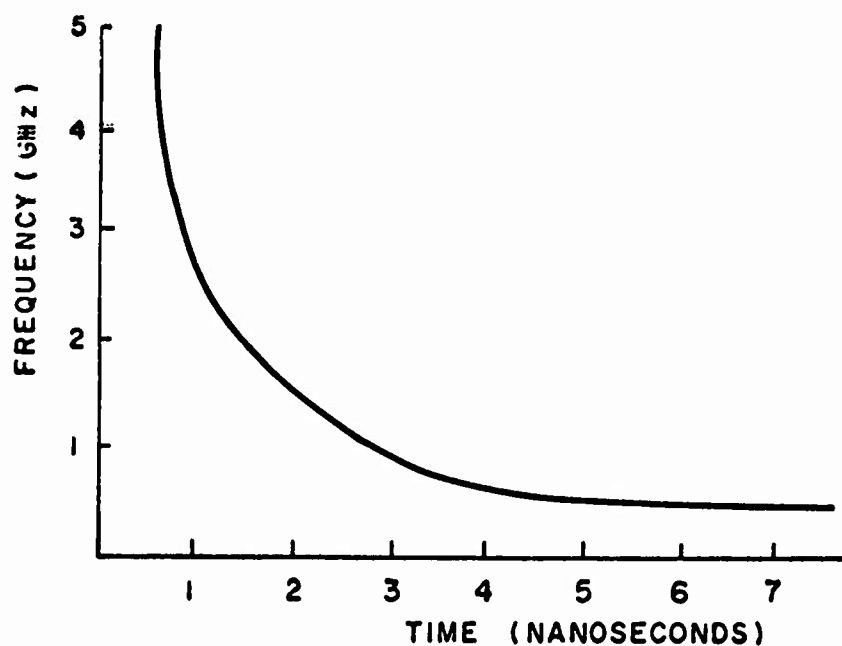
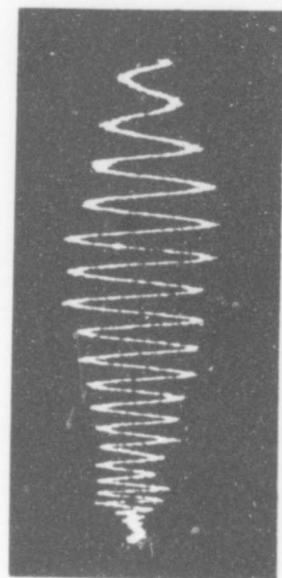
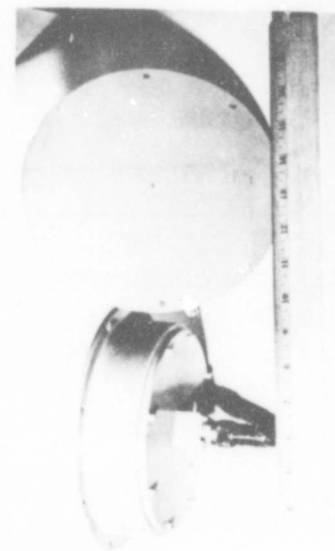


FIGURE 15: Transmitting Transfer Function: Time Vs Instantaneous Frequency Response of Log Periodic Antenna Model #APN-502A



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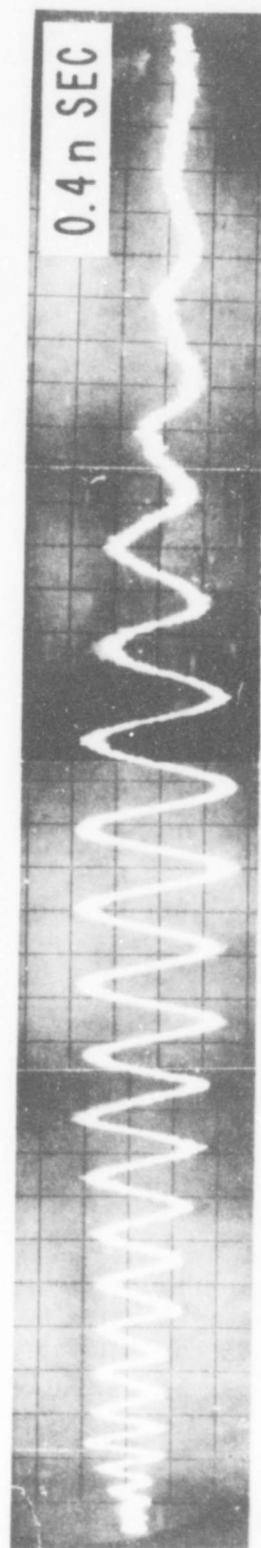


FIGURE 16: Step Response of Two Each Cavity-Backed Spiral Antennas.

## B. Cavity Backed Spiral Measurements:

We shall now consider the transient response of a pair of cavity backed spiral antennas. The particular antennas used were AEL's cavity backed spiral Model ASN 116A with the following published specifications:

1. Frequency: 1.0 GHz to 12.4 GHz
2. VSWR: 2::1
3. 3 dB Beamwidth 70°
4. Gain: 7 dB
5. Polarization: Circular
6. Axial Ratio: 2 dB average
7. Connector: Female Type N

The experimental setup was to feed a step waveform into one antenna and to receive the radiated waveform in an identical cavity backed spiral antenna located four (4) to five (5) feet away. Other than cabling, the only equipment employed was a Hewlett-Packard time domain reflectometer Model 140A/1415A. A d.c. step of 0.2 Volts magnitude with a rise time of approximately 100 picoseconds feeds the transmitting antenna. The receiving antenna feeds the time domain reflectometer, which is used as a sampling scope for reception. It should be noted that in this experiment the dispersion properties of the antenna pair are being measured considering both the transmitting and receiving transfer functions, i.e., this experiment measures the total antenna system transfer function.

The d.c. step response of the cavity backed spiral is seen in Figure 16. Again, a chirp waveform is seen with the frequency monotonically decreasing. The highest "instantaneous frequency" that can be seen is about 6 GHz whereas

the antennas should respond up to 12.4 GHz. This is due to the response of the equipment and more to the fact that the spectrum components of a step function decay with increasing frequency as  $1/f$ . Since the step function response is the integral of the impulse response one may simply take the derivative of Figure 16 to obtain the total receiving and transmitting transfer function. That is, the waveform,  $g(t)$ , of Figure 16 is:

$$g(t) = K \int_{-\infty}^{+\infty} [h_T(t) * h_R(t)] dt \quad (43)$$

The time-frequency plot of the cavity backed spiral antenna is seen in Figure 17. Over the frequency range of 1.2 GHz to 7 GHz there exists a time dispersion of 11.5 nanoseconds. This results in a time-bandwidth product of 66.7. If the measurements included the antennas' response to 12.4 GHz, it is apparent that the time-bandwidth product will be greater than 100.

To further demonstrate the dispersion properties of the spiral antenna, a one gigahertz monocycle (i.e., one cycle of RF) waveform from a Hertzian generator is fed to the transmitting antenna. As seen in Figure 18, the waveform received by the other spiral antenna is again a chirp waveform. From pulse compression theory it is noted that if the complex conjugate of the received waveform is used as the transmitter, then the received waveform will be a monocycle. The complex conjugate waveform would look similar to the chirp pulse of Figure 18 except it would be chirping up in frequency instead of down in frequency.

The above experimental measurements should be considered as preliminary, however, they dramatically demonstrate that dispersion with high time-bandwidth

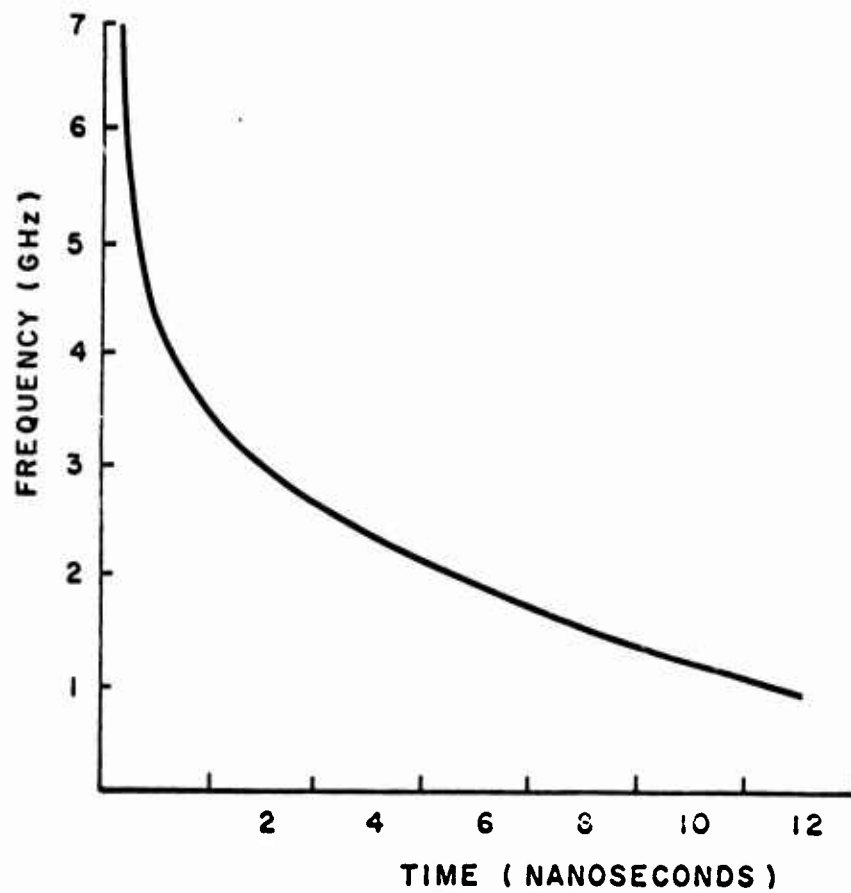


FIGURE 17: Time Vs Instantaneous Frequency Response of Two Cavity-Backed Spiral Antennas Model # ASN-116A.



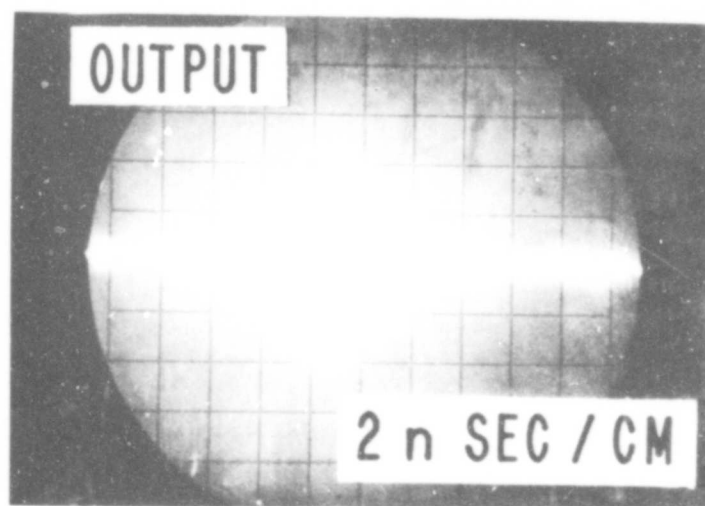
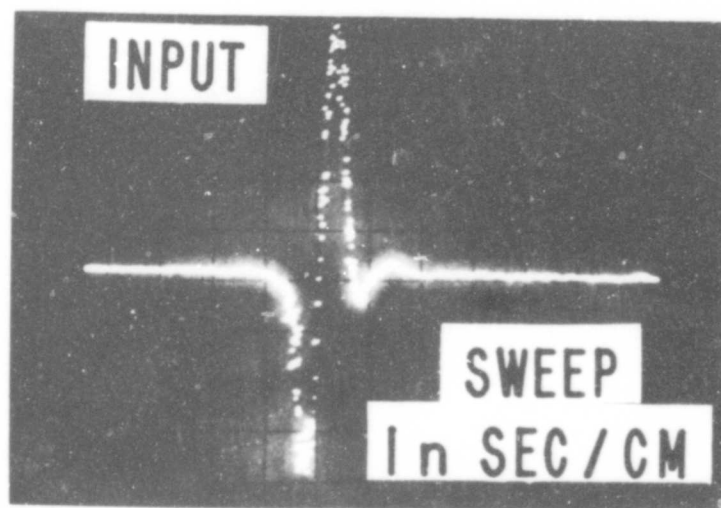


FIGURE 18: Response of Cavity-Backed Spiral Antenna to a 1 GHz Monocycle Waveform.

products do exist in present antennas. It is a bit humorous that the types of antennas tested here are considered to be frequency independent<sup>7</sup>. It appears that if the antenna designer considers dispersion as a primary design factor, antennas can be constructed with very large time-bandwidth products.

## SECTION VI

### SUMMARY AND CONCLUSIONS

It has been shown that enhanced electromagnetic fields can be produced by employing an antenna as a pulse compression network. Antenna systems with natural built-in dispersion can be employed and if properly matched to the transmitter's waveform the electromagnetic peak power density can be increased by the time-bandwidth product. As in pulse compression networks, the pulse width in space is also compressed by an approximate amount equal to the time-bandwidth product.

For the applications of obtaining impulsive waveforms with large energy content this principle has two major advantages. The first is that the time-bandwidth product increases the power density in the E.M. field. The second overcomes the difficulty of maximizing the three antenna parameters of efficiency, directivity and signal fidelity simultaneously. That is, present impulsive antennas with good signal fidelity and directivity suffer from very poor efficiency.<sup>1,8,9,10,11</sup> Dispersive antennas of the type measured above are efficient and the E.M. field will be enhanced by this considerable increase in efficiency.

Another important application is that of obtaining very strong E.M. fields when the transmitter is peak power limited and/or the power handling capability of the transmission line is limited. Since compression occurs spatially, the peak power density can be increased by the time-bandwidth product of the waveform over the limitations set forth above.

It was also shown that compression can be performed both on transmission and reception such as radar applications. In this case the effective BT product is greater than that of radiating only.

Measurements were performed showing well behaved chirped transfer functions for two different antenna systems. It should be noted that the antennas are considered to be frequency independent<sup>7</sup> and were not designed for dispersion properties. If the antenna designer considers the chirp dispersion as a primary design criteria it is expected that antenna systems can be produced with very large time-band products resulting in increased field intensities for impulse antenna systems.

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## APPENDIX A

### DERIVATION OF AN ANTENNA'S TRANSMITTING TRANSFER FUNCTION

The power density,  $P_d$ , at a range  $R$  from an antenna is:

$$P_d = \frac{P_T G}{4\pi R^2} \quad (A-1)$$

Also, the power density and field intensity,  $\Xi(w)$ , are related by:

$$P_d = \frac{\Xi^2(w)}{Z_0} = \frac{\Xi^2(w)}{120\pi} \quad (A-2)$$

where  $Z_0$  is the intrinsic impedance of free space.

Equating the above two equations:

$$\Xi(w) = \frac{\sqrt{P_T G Z_0}}{R \sqrt{4\pi}} \quad (A-3)$$

The antenna's transmitting transfer function is defined as:

$$H_T(w) = \frac{\Xi(w)}{E_T(w)} \quad (A-4)$$

Using Equations A-3 and A-4:

$$H_T(w) = \frac{\sqrt{P_T G Z_0}}{E_T R \sqrt{4\pi}} \quad (A-5)$$

and since:

$$P_T = \frac{E_T^2}{Z_a} \quad (A-6)$$

where  $Z_a$  is the antenna's driving point impedance:

$$H_T (w) = \frac{1}{R} \sqrt{\frac{G Z_0}{4\pi Z_a}} \quad (A-7)$$

Since  $Z_0 = 120\pi$  then:

$$H_T (w) = \frac{\sqrt{30 G(w)}}{R \sqrt{Z_a(w)}} \quad (A-8)$$

## APPENDIX B

### DERIVATION OF AN ANTENNA'S RECEIVING TRANSFER FUNCTION

The power received,  $P_R$ , by an antenna is:

$$P_R = \frac{E_R^2}{Z_b} \quad (B-1)$$

where  $Z_b$  is the antenna's input impedance. Solving for  $E_R$ :

$$E_R = \sqrt{P_R Z_b} \quad (B-2)$$

The power density at the antenna is:

$$P_d = \frac{\Xi^2 (w)}{Z_0} = \frac{\Xi^2 (w)}{120\pi} \quad (B-3)$$

where  $Z_0$  is the intrinsic impedance of free space.

Solving the above equation for the field intensity,  $\Xi (w)$ :

$$\Xi (w) = \sqrt{P_d Z_0} \quad (B-4)$$

The antenna's receiving transfer function is defined as:

$$H_R (w) = \frac{E_R (w)}{\Xi (w)} \quad (B-5)$$



Substituting the expressions of Equations B-2 and B-4 into B-5

$$H_R (w) = \frac{\sqrt{P_R Z_b}}{\sqrt{P_d Z_o}} \quad (B-6)$$

The received power is the product of the power density,  $P_d$ , and the antenna's receiving aperture,  $A$ :

$$P_R = A P_d \quad (B-7)$$

Solving the above equation for  $P_d$ :

$$P_d = \frac{P_R}{A} \quad (B-8)$$

and using this value in Equation B-6:

$$H_R (w) = \sqrt{\frac{A (w) Z_b (w)}{Z_o}} \quad (B-9)$$

Since  $Z_o = 120\pi$  then:

$$H_R (w) = \sqrt{\frac{A (w) Z_a (w)}{120\pi}} \quad (B-10)$$

## APPENDIX C

### DERIVATION OF THE RELATIONSHIP BETWEEN THE TRANSMITTING TRANSFER FUNCTION AND THE RECEIVING TRANSFER FUNCTION

This section will derive the relationship between the transmitting transfer function and the receiving transfer function of an antenna.

Using the factor,  $\Psi$ , the problem is stated:

$$H_T (w) = \Psi (w) H_R (w) \quad (C-1)$$

whereas  $\Psi$  is to be determined. Solving for  $\Psi (w)$ :

$$\Psi (w) = \frac{H_T (w)}{H_R (w)} \quad (C-2)$$

Substituting the value of  $H_T (w)$  from Appendix A and  $H_R (w)$  from Appendix B into Equation C-2 yields

$$\Psi (w) = \frac{H_T (w)}{H_R (w)} = \frac{Z_0}{R} \sqrt{\frac{G (w)}{A(w) Z_a (w) Z_b (w)}} \quad (C-3)$$

and recalling that  $Z_a = Z_b$  for the same antenna

$$\Psi (w) = \frac{Z_0}{Z_a R} \sqrt{\frac{G (w)}{A (w)}} \quad (C-4)$$

Using the value of  $G (w)$  from the relationship:

$$G = \frac{4\pi A}{\lambda^2} \quad (C-5)$$

and  $\omega = 2\pi f = \frac{2\pi C}{\lambda}$  into Equation C-4 yields:

$$\Psi(\omega) = \frac{Z_0 J\omega}{Z_a RC \sqrt{4\pi}} \quad (C-6)$$

The factor  $J$  which appears in Equation C-6 is obtained as a result of taking the square root of Gain ( $G$ ) which is the product of a complex quantity and its conjugate. Consequently, although  $G \propto \omega^2$ ,  $\sqrt{G} \propto \pm J\omega$ . Placing this value in Equation C-1 gives:

$$H_T(\omega) = \frac{Z_0 J\omega H_R(\omega)}{Z_a RC \sqrt{4\pi}} = K J\omega H_R(\omega) \quad (C-7)$$

Taking the Fourier Transform of both sides:

$$h_T(t) = K \frac{d[h_R(t)]}{dt} = K h'_R(t) \quad (C-8)$$

That is to say, for a constant input impedance antenna the transmitting transfer function is the derivative of the receiving transfer function.

The preceding derivation establishes an integral-differential relationship between the transmitting and receiving transfer functions. This relationship has been observed experimentally by a number of investigators who have offered reasonable justification of the results. However, to the best of this author's knowledge, this is the first time that the relationship has been derived from basic radar transmission concepts and formulation.